

# GRADUATION PROJECT

## **AN INVESTIGATION ON DUAL NUMBERS: ALGEBRAIC PROPERTIES, MATRIX REPRESENTATIONS, AND APPLICATIONS**

**Abstract:** Dual numbers, expressed in the form  $a + b\epsilon$  where  $\epsilon^2 = 0$ , provide an algebraic framework for various mathematical and computational problems. This project explores the fundamental algebraic properties of dual numbers and investigates their structural characteristics. We examine the ring properties of dual numbers along with their ideal structures. Then we investigate their representation through  $2 \times 2$  real matrices and some basic properties of this matrix ring. Furthermore, the relationship between dual numbers and Taylor expansions is analyzed and some applications of this expansion on dual numbers are presented. Our study also investigates dual functions and their derivatives. The study concludes with a brief overview of dual numbers in automatic differentiation.