Optimal approximation in Banach spaces

Abstract

Let X be a Banach space with the unit ball $B(X)$ and A be origin symmetric set in X. Kolmogorov n-width $d_n(A,X)$ of A in X is defined as

$$d_n(A,X) = \inf_{L_n \subset X} \sup_{x \in A} \inf_{y \in L_n} \|x - y\|_X ,$$

where L_n is any subspace of X of dimension n. Let

$$b_n(A,X) = \sup_{L_{n+1} \subset X} \sup_{\varepsilon > 0} \{\varepsilon B(X) \cap L_{n+1} \subset A\}$$

be the Bernstein n-width. We present some properties and methods of evaluation of $d_n(A,X)$ and $b_n(A,X)$.