# A GAME THEORETIC APPROACH TO POPULATION DYNAMICS


Fatma Didar Işıldar

The Department of Mathematics
Çankaya University
Ankara, Türkiye


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# Abstract

This graduation project studies epidemic spreading from a game-theoretical perspective and models it as a zero-sum differential game between the virus and society. The goal of the virus in this game is to maximize the number of infected individuals. On the other hand, social entities use control strategies to minimize this number. In this project, epidemic spreading is not considered from a biological perspective.

The model is described by partial differential equations that capture the evolution of the epidemic over time. To solve the equations, the Homotopy Perturbation Method is applied. The solutions obtained can then be used to proceed with numerical solutions.

To determine how well the model fits the actual data, a Python simulation was written and tested using COVID-19 infection data from Turkey, China, Italy, and Germany. The data is analyzed using the Oxford Government Response Tracker Stringency Index for COVID-19 to measure government responses at different points in time. It is clear that control and infections establish an equilibrium that corresponds to the points of stability in the zero-sum game model.

The above findings suggest that zero-sum differential games are an appropriate framework for studying the dynamics of an epidemic, as a strategic struggle can provide insight into the role of government policy during an epidemic.

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**Fatma Didar Işıldar**
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# Chapter 1

# Introduction

Game theory offers a mathematical framework for analyzing scenarios in which the payoff of your choice depends not only on what you do, but also on what others do. Game-theoretic models have been used extensively to analyze strategic behavior in economics, political science, biology, and social sciences since the pioneering work of von Neumann and MorgensternNeumann et al. [1944] . One of these models is the zero-sum games, which represent conflicts where what one side wins is exactly the same as what the other loses.

In the past two decades, classical game theory has been continually generalized to accommodate these dynamic, large systems. Specifically, evolutionary game theory and differential games are designed to study the evolution of strategies with time, seeking equilibrium concepts such as Nash equilibrium and ESS in populations rather than pairsNash [1950]Başar and Olsder [1982]Smith [1983]. These developments have greatly broadened the field of game theory, enabling it to study complex systems that evolve over time and are influenced by many interacting individuals.

The spread of infectious diseases can be considered another example of a complicated system. The process of an epidemic contains constant interaction between two opposing factors: on one side, the spread of a virus, and on the other side, control over this spread. Therefore, the analysis of an epidemic can be considered a strategic struggle, which makes this problem another excellent candidate for a game-theoretic approach. The struggle between the spread and control can be described in a zero-sum game, where an increase in control strategy effectiveness leads to a decrease in the effectiveness of a pathogen spreadMegahed and Madkour [2023].

Based on game theory, especially zero-sum games and their dynamic versions, this project aims to look into the possibility of applying game theory principles to epidemiological data. A real-case application using globally available COVID-19 case numbers will be used to illustrate the possibility of using game theory to make sense of epidemic dynamics. The dynamics of both the virus and control strategies will be described using a strategic game framework in order to apply traditional and evolutionary game theory to real data.

The study builds from theory to implementation by translating mathematical models into computational algorithms that provide simulations showing how strategies and outcomes evolve over time. This work is meaningful in integrating real epidemic data into game-theoretic modeling, demonstrating one way in which abstract mathematical concepts can be adapted to analyze a data-driven problem. In general, the project attempts to bridge theoretical game theory and applied disease modeling in a way that demonstrates zero-sum game frameworks can provide meaningful insight into epidemic dynamics in a mathematically rigorous manner.

# Chapter 2

# Literature Survey And Background

## 2.1 Normal Form Games

A game, in its most elementary form, can be defined in the following way. A game is a situation in which more than one decision-maker makes his or her decision while having information about the possible strategies and payoffs of others. Normal form games provide a mathematical representation of such situations. In the normal form game, the players' strategies that each is faced with, and the payoff function for each player based on the decision of each strategy, are defined. It is presumed that this happens at the same time and that a player does not know the decisions of the other players before he makes his own decisions. Although this can be considered simplistic, it enables analysis. The normal form of game theory can often be applied to a one-shot situation or negotiation. However, in the real world, there exist various forms of processes that relate to societal behavior, which are carried out in a manner that does not take place within a point in time; an issue that becomes a key factor during an epidemic. A normal form game can be formally defined as

$$G = (N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N}),$$

where $N$ denotes the set of players, $S_i$ the strategy set of player $i$, and $u_i$ the corresponding payoff function. This representation does not aim to capture dynamics, but it provides a clean baseline for understanding strategic interaction. In this framework, a strategy profile is denoted by

$$s = (s_1, s_2, \dots, s_n),$$

which determines the outcome of the game. Each player is assumed to be rational and aims to maximize their own payoff function $u_i(s)$. The strategic interaction arises from the fact that the payoff of each player depends not only on their own strategy, but also on the strategies chosen by others. This interdependence is the key feature that distinguishes game-theoretic models from classical optimization problems. Each player chooses a strategy $s_i \in S_i$ in order to maximize their payoff $u_i(s_i, s_{-i})$.

## 2.2 Zero-Sum Games

A zero-sum game represents a special case of a normal form game in which the players have conflicting interests. In a zero-sum game involving two players, an increase in the payoff for one player always causes a decrease in the payoff for the other player. However, this representation suggests that cooperation does not result in mutual benefit. Rather, each player seeks ways to maximize the mitigation of losses. Therefore, the zero-sum game can be considered a representation of conflict, rather than a representation of the problem of coordination. In a two-player zero-sum game, the payoff functions satisfy

$$u_1(s) + u_2(s) = 0,$$

for every strategy profile $s$. As a modeling tool, zero-sum games are highly desirable because they incorporate the idea of strategic equilibrium. With regard to pandemics, the game situation is quite simple: the spread of the virus translates to a loss for the community, and the effectiveness of control measures implies the loss of capacity by the virus to spread. Although this game is quite simplistic, it portrays the essence of competition. In the case of zero-sum games, the strategic problem can be restated as a minimax optimization problem. One player aims to maximize the payoff, and the other aims to minimize the payoff. This situation gives rise to the minimax principle.

$$\max_{u \in U} \min_{v \in V} J(u, v) = \min_{v \in V} \max_{u \in U} J(u, v)$$

Under suitable convexity and compactness conditions, von Neumann's minimax theorem guarantees the equality aboveNeumann et al. [1944].

## 2.3 Nash Equilibrium and Saddle Points

In a strategic context, a question that would naturally arise would be the existence of a stable state where no player has any reason to alter its play. This is captured through the definition of a Nash equilibrium. Formally, a Nash equilibriumNash [1950] in a zero-sum game is a strategy pair (u*, v*) such that

$$u^* \in \arg\max_u J(u, v^*),$$
$$v^* \in \arg\min_v J(u^*, v).$$

In zero-sum games, the Nash equilibrium involves saddle points. In this respect, the strategy of one player is optimal to the strategy of the other player at the saddle point. Moreover, there are no advantages if they change their strategy. This concept is mathematically modeled as an inequality in terms of payoffs, although what it represents is simple: both sides are doing their level best in the circumstances that exist. Such an equilibrium in epidemic control could be described as the point at which events are in balance in terms of both control and spread. Too early relaxation of control measures, and an epidemic equilibrium tips in favor of the epidemic. Excessively strict measures may lead to severe social and economic costs, with significant social and economic losses. A saddle point equilibrium satisfies

$$J(u, v^*) \leq J(u^*, v^*) \leq J(u^*, v).$$

At this point, neither side can improve its outcome through unilateral deviation.

## 2.4 Differential Games

Static models will not be appropriate for cases of dynamic changes, since the strategies and results are always varying over time. A differential game representsBaşar and Olsder

[1982] an extension of the static games concept, since for differential games, the state of the system described is affected by differential equations. In the differential game, the players influence the dynamic system according to the time-dependent control strategyKaraman [2010]. In the decision that relies upon the moment in time, the consequences appear in the current instance; however, more significantly, in the future. It is for this reason that the differential game is useful in the modeling of epidemic spreading cases because the levels change gradually. When differential games are set up within a context of a zero-sum game, the idea of the saddle point can be extended to the dynamic system. These time-specific points of equilibrium form the basis for the computational model that will be developed in the next chapter. In differential games, the evolution of the system is governed by a state equation that depends on the control strategies of the players.

$$\dot{x}(t) = f\big(x(t), u(t), v(t), t\big), \text{ with initial condition} \qquad x(0) = x_0.$$

$$J(u, v) = \int_0^T L\big(x(t), u(t), v(t), t\big)\, dt.$$

The objective of each player is to optimize this functional over the time horizon [0,T].

# Chapter 3

# Zero-Sum Game, Population Dynamics, And Model Comparison

## 3.1 The Strategic Duel

A Zero-Sum View of the Pandemic. In contrast to a general game-theoretic notion described within the previous chapter, this section dwells on a particular framework used for epidemic modeling. Contrary to the biological explanations of the pandemic, a pandemic is not only a natural phenomenon but is a product of the interaction of two conflicting forces. From this viewpoint, the pandemic can be considered as an endless fight between the disseminating disease and the collective efforts of society. The emergence of the COVID-19 pandemic is represented as a zero-sum differential game based on the theory devised by Megahed & Madkour Megahed and Madkour [2023]. In a scenario of a zero-sum game, the objective of one player is considered as the negation of the other.

For instance, the objective of Player 1 or the Pathogen would be to maximize the existence triggered by the outcome of the game. Conversely, the objective of Player 2 or Humanity would be to limit the aforementioned objective. This dynamic is essentially a zero-sum game. The growth of the virus, for instance, through infections or hospitalization, essentially captures the loss triggered within society because of the aforementioned growth. Contrary to the static modeling of the aforementioned games, this dynamic continues to evolve over time. Each change triggered by the behavior of the aforementioned virus would essentially be opposed by society through dynamic adjustments and changes. This would essentially mean that the value of the aforementioned game or the objective at a point of time would be triggered by the aforementioned dynamic.

## 3.2 The Mathematics of Spread Governing through PDEs

In order to capture the dynamic nature of the dissemination process in the population, it is not sufficient to describe it using numbers alone, and therefore, Partial Differential Equations will be employed for its description. Based on the article on the topic by Megahed and MadkourMegahed and Madkour [2023], instead of describing the virus using the fixed object concept, it is indeed more appropriate to describe it as a process that changes over interactions of time and strategies. In this model, the diffusion of the virus is described using the Infinity Laplacian operator $\Delta_\infty$. The Infinity Laplacian operator $\Delta_\infty$ is a mathematical tool for describing the diffusion of the virus through the network formed by the structure of modern society. The governing equation of the system is given by

$$\frac{\partial \Phi}{\partial t} = \Delta_\infty \Phi(x, y, t) + f(x, y, t, u, v).$$

Here, $\Phi$ represents the infection potential within the population. The term $\frac{\partial \Phi}{\partial t}$ describes how this potential changes over time, indicating whether the spread of the virus is accelerating or slowing. The function $f(u, v)$ models the interaction between human control measures $u$ and the virus's replication dynamics $v$, reflecting the strategic feedback between the two sides.

## 3.3 Mathematical Adaptation for Computational Implementation

To make the prepared model suitable for numerical analysis, the analytical solution obtained using the Homotopy Perturbation Method (HPM)He [1999] is converted to a computational form. The resulting series solution used in the simulation is given by:

$$u(x,t) = e^{-x} + e^{-3x}t + \frac{15}{2}e^{-5x}t^2 + \frac{217}{2}e^{-7x}t^3 + \frac{3429}{4}e^{-9x}t^4$$

Within the computational framework, each model variable is mapped to a specific dataset variable for simulation and analysis.

- **Strategic resistance (x):** mapped to the normalized Stringency Index. Higher values of x correspond to stronger intervention policies, which reduce the magnitude of the exponential terms and indicate more effective suppression.
- **Time (t):** represents the progression of pandemic days and is normalized to remain within the convergence range of the series solution.
- **Outcome u(x,t):** represents the theoretical infection risk produced by strategic interaction, rather than a direct epidemiological count. This interpretation allows the abstract mathematical solution to be meaningfully connected to observable policy and infection trends.

## 3.4 Finding the Saddle Point: The Quest for Balance

In a lot of strategy conflicts, there exists a critical zone where a player has no way of actually improving his position without finding himself in a worse position than before. This critical zone in game theory exists at a point known as the Saddle Point Equilibrium. According to Megahed and Madkour Megahed and Madkour [2023], this equilibrium is defined by the inequality

$$J(u^*, v) \leq J(u^*, v^*) \leq J(u, v^*).$$

This represents the state at which an equilibrium is formed between the spread of the virus and the level of government response. At the equilibrium state, it can be noted that the level of response must be sufficiently high to prevent the healthcare system from being overwhelmed by patients, but at a level that would not significantly impact the social-economic system. Any level of response that is outside the equilibrium state but before the optimal response level ($u < u^*$) would render the virus an advantage.

## 3.5 Evolution in Action: Replicator Dynamics and Real-World Data

Replicator dynamics from evolutionary game theory are employed to link the theoretical model with observable behavior. In this framework, strategies evolve over time analogously to biological traits, with their prevalence increasing or decreasing based on relative success. This process is given by the replicator equation

$$\dot{x}_i = x_i\big[f_i(x) - \phi(x)\big].$$

This modeling approach enables assessments of the impact of various societal behavior patterns—e.g., social distancing or continued usual activity—on their prevalence over time, based on changing conditions. According to the ever-changing viral strategies in their own right, societal behaviors in response to the virus will also change in an equal manner. By accounting for real-world factors such as the Oxford Stringency Index and the number of daily reported cases, this modeling method connects mathematical concepts of pandemic patterns to the pandemic experience.

# Chapter 4

# Computational Analysis and Simulation Results

Unlike the preceding chapters, where exemplary emphasis was placed on the theoretical background and mathematical modeling, this chapter will concentrate on the computational process of implementing the presented framework as well as discussing the results obtained. The objective of this chapter shall be to investigate whether the zero-sum differential game model developed previously can be related to real-world epidemic outbreaks.

## 4.1 Computational Implementation

To validate the theoretical framework, a Python-based computer simulation was developed. The mathematical series solution from the previous chapter was applied to real-world data from the Oxford COVID-19 Government Response Tracker (OxCGRT)Hale et al. [2021]. The simulation utilized globally available COVID-19 data from the Oxford COVID-19 Government Response Tracker (OxCGRT). This dataset provides harmonized indicators for multiple countries, enabling the application of the differential game theory framework at a global scale rather than being limited to a single country. For implementation purposes, country-level time series are treated independently within the same modeling structure, ensuring that the analysis remains consistent with the assumptions of the zero-sum differential game. Normalization: The "Stringency Index" (representing Player 2's strategy $u$) and "Confirmed

Cases" (representing Player 1's payoff $v$) were normalized to the $[0, 1]$ interval to align with the mathematical domain of the differential game model. Series Application: The HPM seriesHe [1999] solution was calculated iteratively for each time step, generating a "Theoretical Infection Risk" curve based on the government's daily stringency level.

## 4.2 Analysis of Results

The comparative visualization generated by the model overlays the actual infection spread (empirical data) against the theoretical risk calculated by the zero-sum differential game.

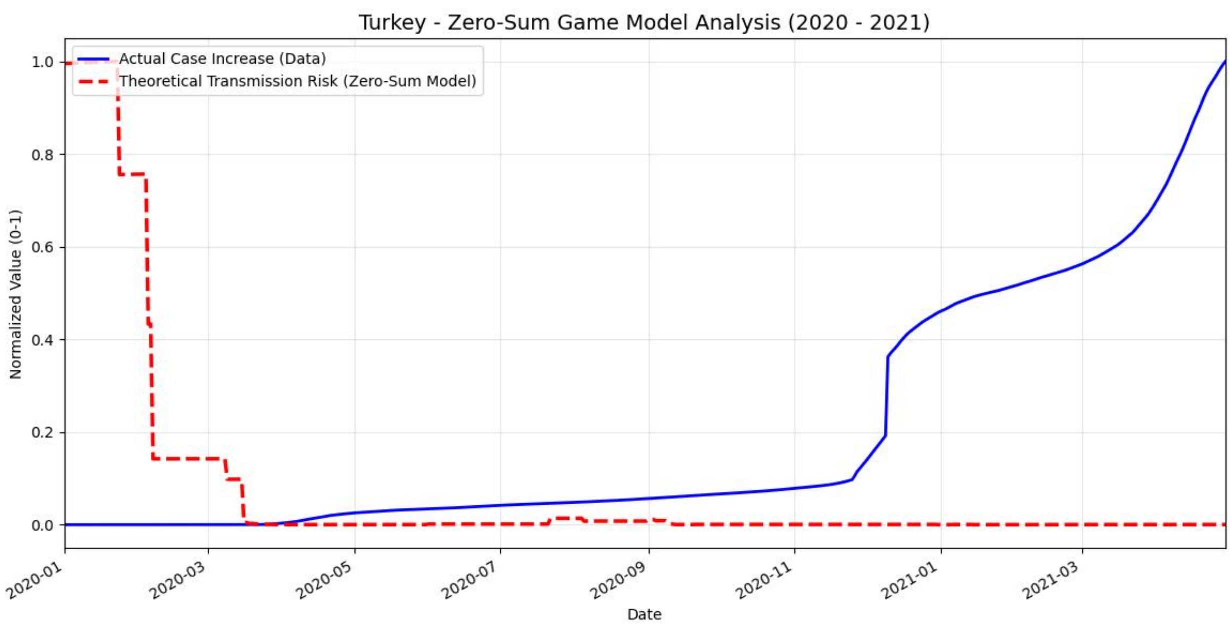


Figure 4.1: Comparison of Actual Infection Spread vs. Theoretical Risk for Turkey (Zero-Sum Game Model)

The analysis of the resulting graph reveals several key insights regarding the "Saddle Point" equilibrium:

- **Equilibrium Phases:** During periods when the government response (Stringency Index)Hale et al. [2021] remained high, the theoretical risk curve and the actual case growth curve converged and behaved similarly. The implication is that a Saddle Point Equilibrium, as discussed in section 2.4, had been reached temporarily, since neither the virus nor society could ultimately maximize or suppress the risk posed by the virus without total seclusion.

- **Strategic Gaps:** Differences between the theoretical risk and the actual cases revealed the time intervals during which the equilibrium has been disrupted. This is where the strategic advantage arose because the control strategies (u) relaxed ahead of the reduction in the viral load (v), favoring the viral infection further, resulting in the next waves. It reinforces the "tug-of-war" dynamic associated with a zero-sum game illustrated in Chapter 2.

### 4.2.1 Comparative Case Study: China

Looking into the Chinese case, there are a number of characteristic aspects related to the Saddle Point equilibrium, mainly caused by the quick actions taken in the early stages.

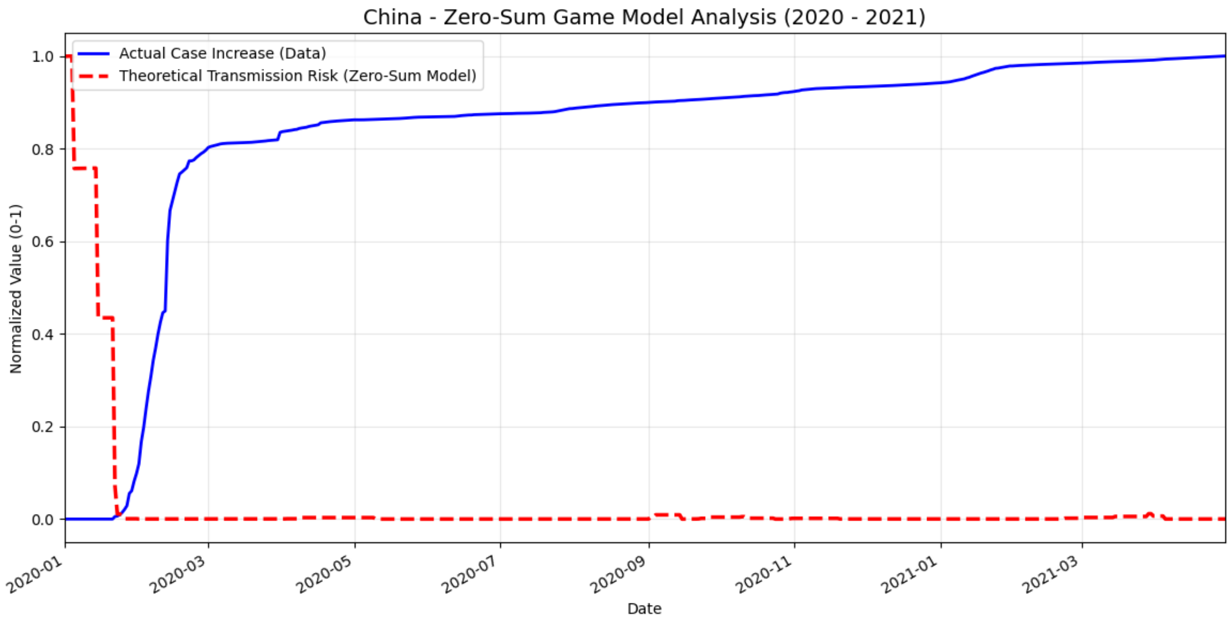


Figure 4.2: Comparison of Actual Infection Spread vs. Theoretical Risk for China (Zero-Sum Game Model)

- **Equilibrium Phases:** However, unlike the slow convergence process, there is an immediate stabilization process in the Chinese scenario. With the sharp increase in the Stringency IndexHale et al. [2021], there was an immediate drop in the theoretical transmission risk to zero. After the initial peak, there was an immediate plateau in the actual cases, showing that there was zero growth in the number of infections.

- **Strategic Gaps:** There has been a visible gap between the theoretical value and the number of reported cases because of the accumulation of infections prior to the effective control measures. However, as the control measures were put in place in their entirety, the gap was prevented from increasing. This illustrates that the government intervention of Player 2 has been successful in stabilizing the game environment so as not to allow the advantage of the Virus growth because the game environment has been brought to the Saddle point.

### 4.2.2 Comparative Case Study: Italy

The Italian case provides evidence that it is not simple to preserve a balance as the growth rate becomes faster due to the virus's spread.

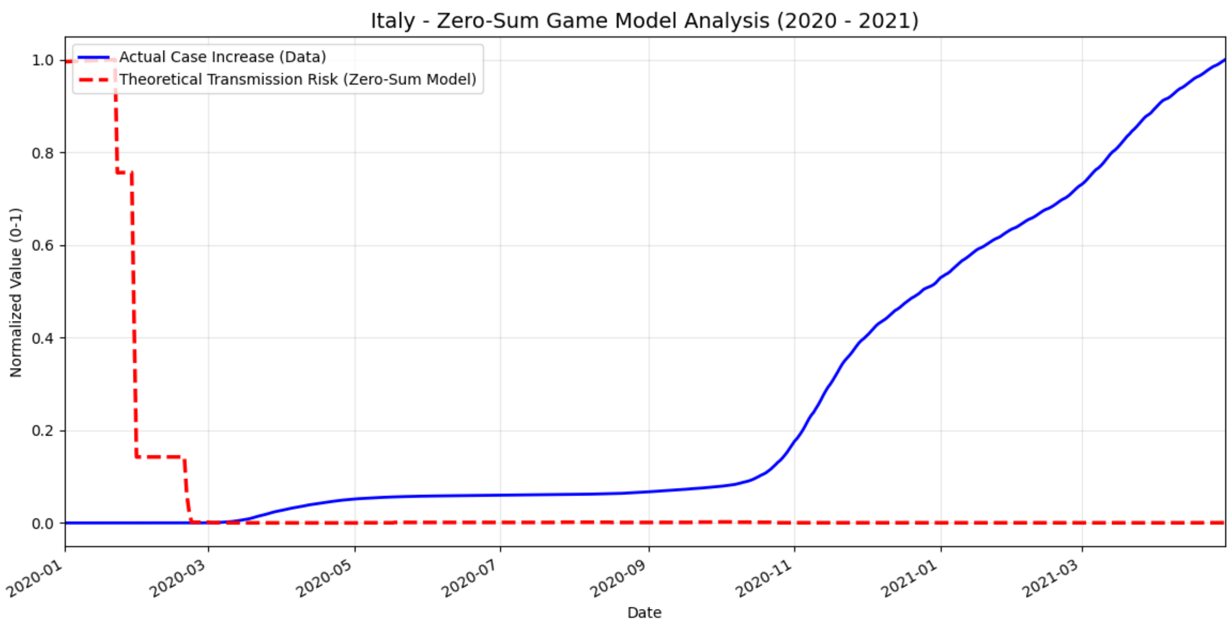


Figure 4.3: Comparison of Actual Infection Spread vs. Theoretical Risk for Italy (Zero-Sum Game Model)

- **Equilibrium Phases:** During the first wave, the government had stepped up its efforts, but the reported cases continued to increase, albeit for a short while, before the rates declined. However, this confirms that though effective mechanisms had been employed to control the spread, the timing was late considering the massive spread of the virus. As such, the equilibrium phase took longer to attain.
- **Strategic Gaps:** In this case, there is a strategic gap if the risk transmission is about to decrease, but the number of infections has yet to slow down. Thus, this strategic gap illustrates the delay between the application of these strategies and the generation

of their resultant outcomes. From a strategic perspective, this imbalance was formed by the fact that the applied strategies were mainly reactive instead of being preventive. After this imbalance is formed, it is difficult to maintain stability.

### 4.2.3 Comparative Case Study: Germany

The data analysis from the graph for Germany shows the impact of technological intervention in changing the dynamics concerning the "Saddle Point" as follows:

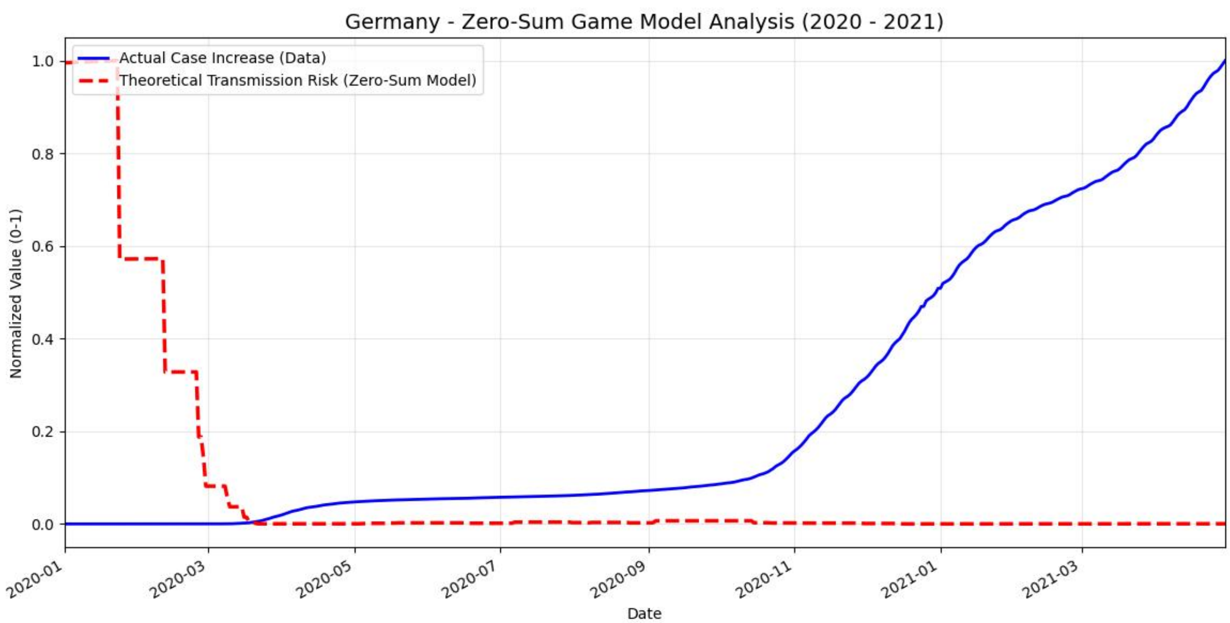


Figure 4.4: Comparison of Actual Infection Spread vs. Theoretical Risk for Germany (Zero-Sum Game Model)

- **Equilibrium Phases:** There is a visible divergence in this equilibrium, unlike the other scenarios. The equilibrium phases in this case are defined not only by the restrictions imposed (Stringency Index)Hale et al. [2021] but are further characterized by the necessity of the vaccine. This shows that the Saddle Point Equilibrium shifted from dependence on physical isolation to a new dependence on biological immunity, allowing Player 2 (Society) to control the spread without indefinite isolation.

- **Strategic Gaps:** The gaps that appeared in the latter stages clearly indicate that there has been a shift in the parameters of the game. With the distribution of the vaccine developed in GermanyPolack et al. [2020], the "tug-of-war" situation has shifted, as the control strategy ($u$) has become more efficient. The existence of these strategic gaps symbolizes a transition from a containment game to an eradication game.

# Chapter 5

# Conclusion

The aim of this research project was to explore whether the intricate dynamics of a pandemic could be abstractly represented and solved using game theory as a strategic conflict. By incorporating key ideas from Normal Form and Differential Games, along with the specific Zero-Sum PDE approach of Megahed and MadkourMegahed and Madkour [2023], this research established a connection between theoretical and practical analyses of a pandemic situation.

The results lead to three main conclusions:
**Validity of the Zero-Sum Framework:** Modeling the pandemic as a zero-sum game is a powerful approximation. The antagonistic relationship between viral spread and control measures creates a competitive structure where "saddle point" equilibria can be identified and analyzed.

**Importance of Time Dynamics:** Static game models fail to capture the evolving nature of an epidemic. The use of differential games proved essential for understanding how strategies must adapt continuously over time rather than be made as one-off decisionsBaşar and Olsder [1982].

**Data-Driven Insight:** The computational application demonstrated that theoretical math models are not just abstract constructs but can track real-world epidemiological trends when fed with accurate policy dataHale et al. [2021].

In summary, this project demonstrates that game theory offers a robust analytical lens for epidemiology. While a zero-sum assumption is a simplification of biological reality, it provides a mathematically rigorous framework for quantifying the relentless strategic struggle between a pathogen's drive to replicate and humanity's effort to survive.

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# Appendix A

# Appendix

### Python code for zero-sum game of population dynamics

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import sys

FILE_NAME = r"OxCGRT_compact_national_v1.csv"
SELECTED_COUNTRY = 'Turkey'

def model_prediction(x, t):
    u0 = np.exp(-x)
    u1 = np.exp(-3*x) * t
    u2 = (15/2) * np.exp(-5*x) * t**2
    u3 = (217/2) * np.exp(-7*x) * t**3
    u4 = (3429/4) * np.exp(-9*x) * t**4
    return u0 + u1 + u2 + u3 + u4

try:
    print(f" Reading file...")
    df = pd.read_csv(FILE_NAME, low_memory=False)
except FileNotFoundError:
    print(f"\n ERROR: File not found! Check the path: {FILE_NAME}")
    sys.exit()

found_columns = [col for col in df.columns if 'StringencyIndex' in col]
```

```python

if not found_columns:
    print("\n ERROR: No data containing 'StringencyIndex' found in the file!")
    print("Available Columns:", df.columns.tolist())
    sys.exit()

S_COL = found_columns[0]
print(f" Automatically Detected Column: '{S_COL}'")

df['Date'] = pd.to_datetime(df['Date'], format='%Y%m%d', errors='coerce')

df_analysis = df[
    (df['CountryName'] == SELECTED_COUNTRY) &
    (df['Date'] <= '2021-04-30')
].copy()

if df_analysis.empty:
    print(f"\n ERROR: No data found for '{SELECTED_COUNTRY}'.")
    sys.exit()

df_analysis = df_analysis.sort_values('Date')
df_analysis = df_analysis.dropna(subset=[S_COL, 'ConfirmedCases'])

print(" Performing calculations...")

days = (df_analysis['Date'] - df_analysis['Date'].iloc[0]).dt.days
t_val = days / days.max() * 0.1  # Normalize time
x_val = df_analysis[S_COL] / 10.0  # Use the newly found column

cases_actual = df_analysis['ConfirmedCases']
cases_norm = (cases_actual - cases_actual.min()) / (cases_actual.max() - cases_actual.min())

model_results = np.array([model_prediction(x, t) for t, x in zip(t_val, x_val)])
model_norm = (model_results - model_results.min()) / (model_results.max() - model_results.min())

```

```
print(" Plotting graph...")
plt.figure(figsize=(14, 7))

plt.plot(df_analysis['Date'], cases_norm, label='Actual Case Increase (Data)', color='blue', linewidth=2)

plt.plot(df_analysis['Date'], model_norm, label='Theoretical Transmission Risk (Zero-Sum Model)', color='red', linestyle='--', linewidth=2.5)

plt.xlim(df_analysis['Date'].min(), pd.Timestamp('2021-04-30'))
plt.title(f"{SELECTED_COUNTRY} - Zero-Sum Game Model Analysis (2020 - 2021)", fontsize=14)
plt.xlabel("Date")
plt.ylabel("Normalized Value (0-1)")
plt.legend(loc='upper left')
plt.grid(True, alpha=0.3)
plt.gcf().autofmt_xdate()

plt.show()
print(" Process completed.")
```