

# An Investigation of Matrices over the Ring of Dual Numbers

## Abstract

The ring of dual numbers is a commutative extension of real numbers, defined as  $\mathbb{D} = \{a + b\varepsilon \mid a, b \in \mathbb{R}, \varepsilon^2 = 0\}$ . Due to the existence of zero divisors, linear algebra over dual numbers have different behaviors compared to classical vector spaces. In this project, we investigate certain algebraic properties of matrices with dual number entries. We first define the set of  $n \times n$  dual matrices and examine fundamental matrix operations. We then analyze the conditions under which a dual matrix can be invertible. Furthermore, we investigate the determinant and trace of dual matrices and examine methods for solving systems of linear equations on the ring of dual numbers  $\mathbb{D}$ .