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Solve It

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For two vectors $\vec{u}, \vec{v} \in \mathbb{R}^n$ the inner (dot) product is given by

$$\langle \vec{u}, \vec{v} \rangle = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n,$$

where $\vec{u} = (u_1, u_2, \ldots, u_n)$ and $\vec{v} = (v_1, v_2, \ldots, v_n)$. Then Cauchy - Schwarz inequality suggests the following inequality:

$$|\langle \vec{u}, \vec{v} \rangle| \leq \|\vec{u}\| \, \|\vec{v}\|,$$

where $\|\vec{u}\| = \sqrt{u_1^2 + u_2^2 + \cdots + u_n^2}$.

Using this inequality, show that:

1. for $u_1 + u_2 + \cdots + u_n = 1$, $u_1^2 + u_2^2 + \cdots + u_n^2 \geq \frac{1}{n}$
2. $(u_1 + u_2 + \cdots + u_n)^2 \leq n \left(u_1^2 + u_2^2 + \cdots + u_n^2\right)$