ÇANKAYA UNIVERSITY

Department of Mathematics

Solve It

- 1 -

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Question: The following second-order Sturm–Liouville differential equation

$$-(p(x)y')' + q(x)y = \lambda w(x)y \,,$$

can be introduced as the following matrix differential equation

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ py' \end{bmatrix}' = \left(\lambda \begin{bmatrix} w & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -q & 0 \\ 0 & p^{-1} \end{bmatrix} \right) \begin{bmatrix} y \\ py' \end{bmatrix}$$

where p, q, w are some real-valued functions such that $p(x) \neq 0$ on a given interval. Find a similar matrix differential equation representation for the following second-order Sturm–Liouville differential equation with distributional potentials

$$-\left(p(x)\left[y' + s(x)y\right]\right)' + p(x)s(x)\left[y' + s(x)y\right] + q(x)y = \lambda w(x)y \,,$$

where p, q, s, w are some real-valued functions such that $p(x) \neq 0$ on a given interval.