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Department of Mathematics

Solve It

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Write the statement of the Theorem(below) after reading the proof.

Theorem..?

Proof.

Throughout, p and q are relatively prime natural numbers, and a and b denote non-negative integers.
Suppose, for contradiction, that

$$pq - p - q = pa + qb.$$

Rewrite this equality as

$$pq = p(a+1) + q(b+1).$$

Since p and q are relatively prime, this equality shows that p divides $b+1$ and q divides $a+1$. Moreover, $b+1 \neq p$, since otherwise we would get $a < 0$. Similarly, $a+1 \neq q$. Hence $p < b+1$ and $q < a+1$, that is, $b > p-1$ and $a > q-1$. Using these inequalities, we obtain

$$\begin{aligned} pq - p - q &= pa + qb \\ pq - p - q &> p(q-1) + q(p-1) \\ pq - p - q &> 2pq - p - q \\ pq &> 2pq, \end{aligned}$$

which is impossible. This contradiction completes the proof.