ÇANKAYA UNIVERSITY

Department of Mathematics

Solve It

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Question 1: Prove or disprove the following statement.
There exists a real-valued continuous function f satisfying the following conditions:

1) For all $x \in dom(f)$, $f(x) \neq 0$.

2) There exists $x_0, x_1 \in dom(f)$, $f(x_0) < 0$ and $f(x_1) > 0$.

Question 2: Prove or disprove the following statement.
For all $a, b \in \mathbb{R}$ such that $a < b$, there exists a continuous function $f : [a, b] \to \mathbb{R}$ satisfying the following conditions:

1) There exists $x_0, x_1 \in [a, b]$ such that $f(x_0) < 0$ and $f(x_1) > 0$.

2) For all $x \in [a, b]$, $f(x) \neq 0$.

Question 3: Prove or disprove the following statement.
Let $f : [a, b] \to \mathbb{R}$ be a continuous function for some $a, b \in \mathbb{R}$ with satisfying the following conditions:

1) If there exists $x \in [a, b]$ such that $f(x) = 0$, then $x = a$ or $x = b$.

2) There exists $x \in [a, b]$ such that $f(x) > 0$.

Then, $f(x) > 0$ for all $x \in (a, b)$.